A DIFFUSION-BASED MODEL FOR INVERSE ELECTROCARDIOGRAPHY (ECGI)

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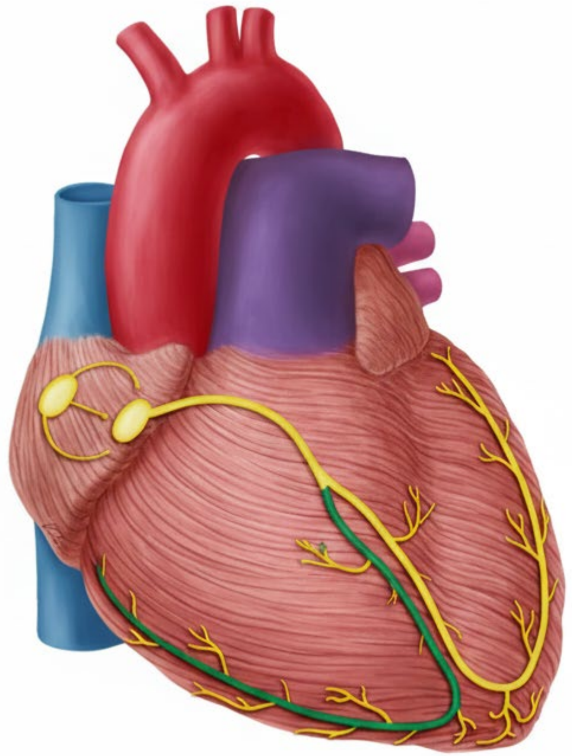
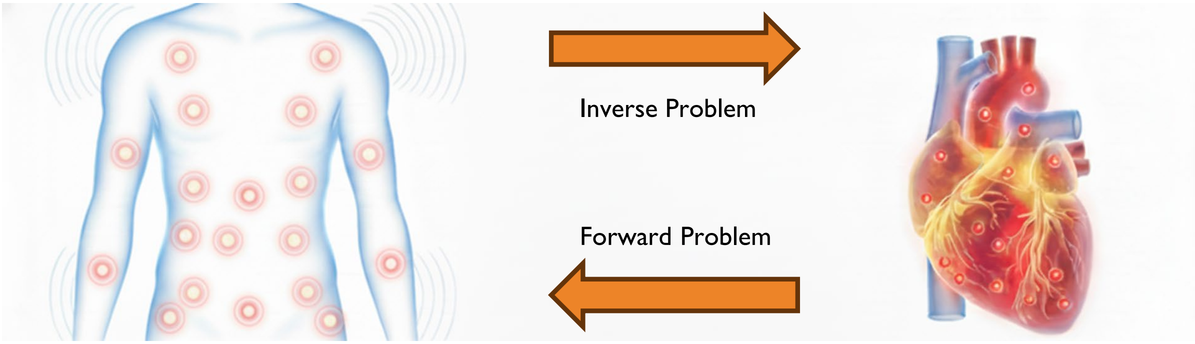
MOTIVATION

- Cardiovascular diseases are one of the major causes of death. Arrhythmias contribute significantly to heart failure and sudden cardiac death
- Understanding arrhythmia mechanisms **requires high-resolution mapping of electrical activity on the heart surface**.
- Current techniques are invasive, limited to a single procedure and performed under sedation, making them unfeasible.
- Clinicians use non-invasive body surface recordings. However, they lack the detail needed to understand complex patterns in the heart.
- **There is a need for a non-invasive and patient-friendly alternative** to guide diagnosis and therapy.



WHAT IS ECGI?

- ECGI (Electrocardiographic Imaging) reconstructs heart surface electrical activity from body-surface potential maps (BSPM)
- Combines:
  - High-density torso electrodes
  - Patient-specific CT/MRI geometry
  - Computational modeling
- Aims to replace or reduce the need for invasive intracardiac mapping in clinical procedures.



MATHEMATICAL MODELING

- Let

  $x \in R^{N_h}$ = heart-surface potentials

  $y \in R^{N_t}$ = torso / body-surface potentials (ECG/BSPM)

- The forward problem (heart ⟶ torso) is modeled as:

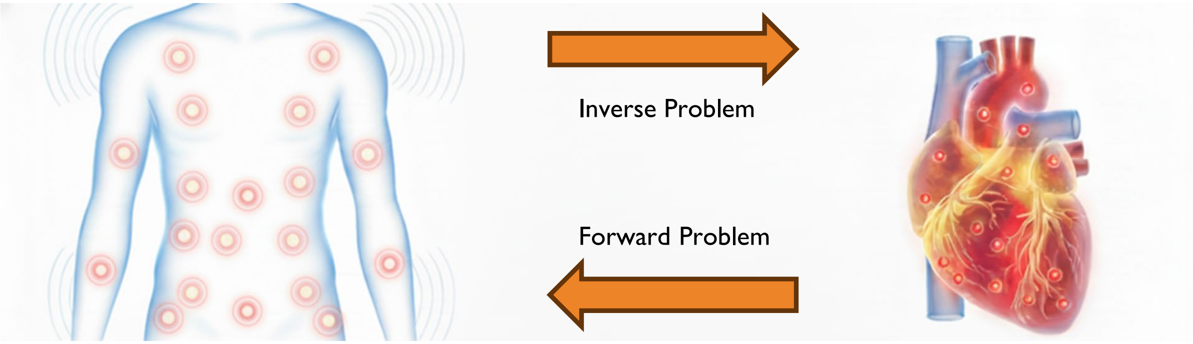
$$y = G\,x + \varepsilon$$

where

G = transfer matrix from heart to torso (computed solving Laplace's equation)

ε = measurement noise, modeling errors, geometry errors, etc.

- The **inverse ECGI problem** is: Given y and G, recover x.



- Classical approach: Tikhonov regularization

$$\hat{x} = \arg\min_x |Gx - y|_2^2 + \lambda |Lx|_2^2$$

- First term: data fit (match torso potentials)
- Second term: smoothness / regularization (e.g., L spatial derivative)
- Problems:

  G is ill-conditioned ⇒ inverse is ill-posed

  Small noise ⇒ large reconstruction errors

  Many possible heart-surface maps can produce the same torso potentials

Classical methods assume a very simple prior:

$x \sim \mathcal{N}(0, \Sigma)$

The reality is that hearts show highly structured patterns!

Machine learning (deep nets) lets us learn this prior from data

Learn a mapping f_θ: $y \mapsto \hat{x}$ where f_θ is a neural network with parameters θ

WHY A DIFFUSION MODEL-BASED APPROACH?

- Score based diffusion models are SOTA across many domains
- They gradually add noise to real data (forward process) and learn to reverse this corruption step by step (reverse process), transforming pure noise into a realistic sample
- In our problem, we want to learn the full distribution

$$p_\theta(x \mid y)$$

Given torso measurements y, there might be many plausible heart patterns x. Let's learn the whole distribution, not just one point estimate.

- This is the idea behind a generative model, you train a model so you can sample:

$$x_0 \sim p_\theta(x \mid y)$$

- Using Bayes Theory:

$$p(x \mid y) \propto p(y \mid x)\, p(x)$$

$p(y \mid x)$ = likelihood from physics: $y \approx Gx + \varepsilon$

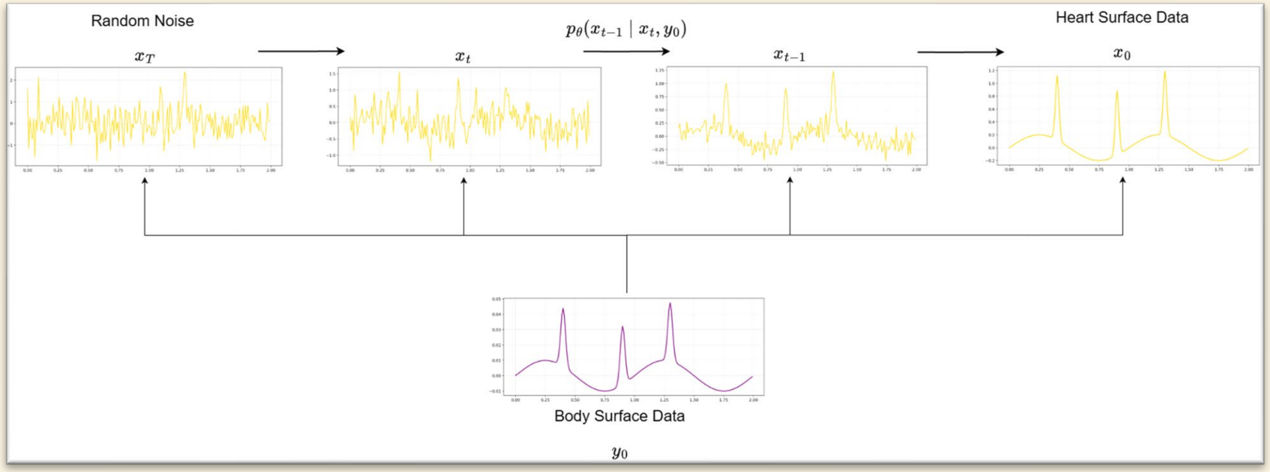
$p(x)$ = prior = how does heart maps look like before seeing the data

- We replace the simple Gaussian by a learned prior:

$$x \sim N(0, \Sigma) \Rightarrow \; p(x) \approx p_\theta(x)$$

- We take a heart-surface signal x_0 and gradually add Gaussian noise until it becomes almost pure noise x_T.
- We train a neural network to reverse this corruption, step by step, conditioned on the body-surface data y_0.
  When generating heart data, we start from random noise and iteratively denoise using the torso signal y_0 as guidance:

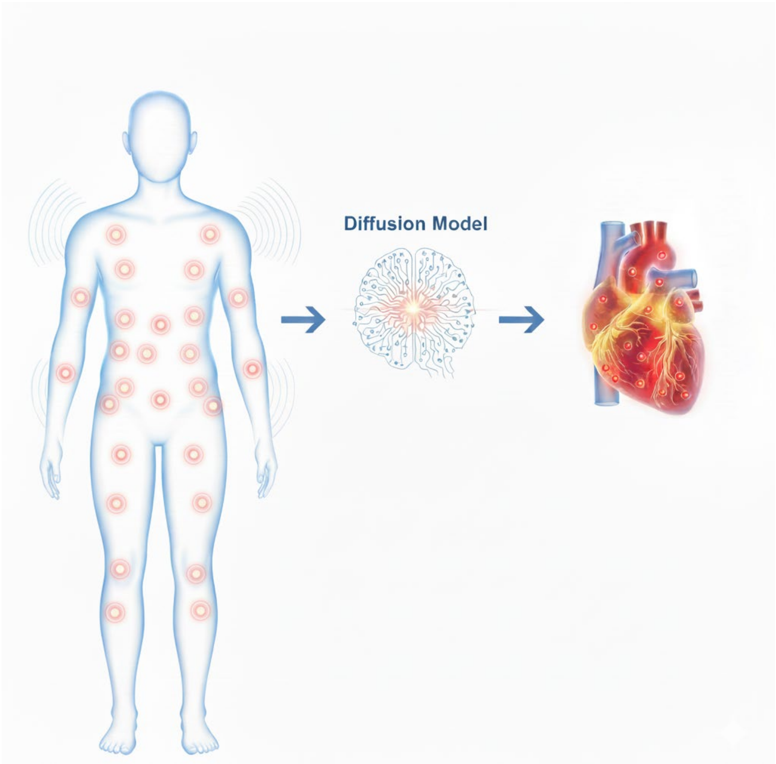
$$x_T \to x_{T-1} \to \cdots \to x_0$$



PROJECT OUTCOME

- Accurate reconstruction over time of heart-surface potentials at multiple locations in the heart.
- Calibrated uncertainty estimates through sampling.
- Evaluation with MAE, MSE, CRPS and correlation metrics.

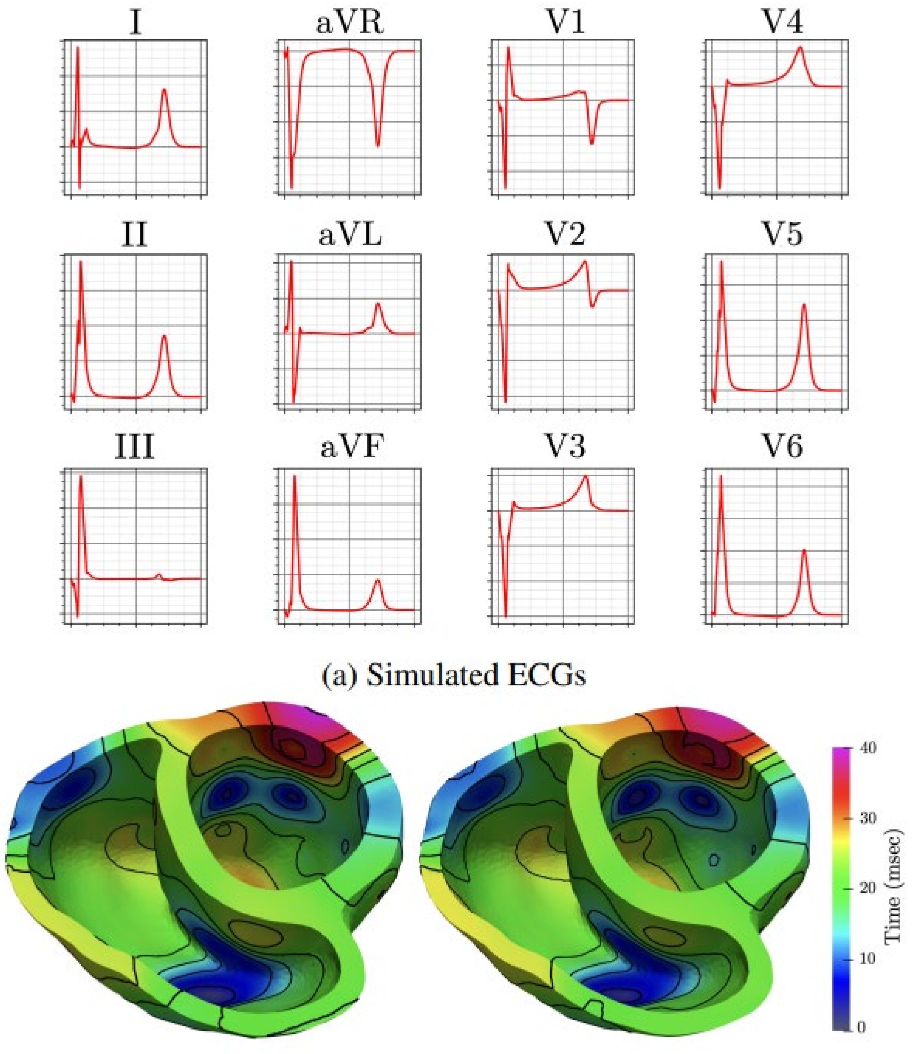
The project aims to make noninvasive heart-surface electrical mapping more reliable!



DATASET

Lawrence Livermore National Laboratory (LLNL) synthetic dataset.

- Over16,000 high-fidelity organ-level cardiac simulations
- Simulated over realistic bi-ventricular geometries for 500 ms of simulation time.
- 12-lead ECG signals ($y \in \mathbb{R}^{12\times500}$)
- Full transmembrane voltage maps ($x \in \mathbb{R}^{75\times500}$) covering 75 heart surface points
- Used to reconstruct activation maps in previous work
- We want to reconstruct the full spatiotemporal voltage waveforms (electrical potentials over time at many heart locations).



(a) Simulated ECGs

(b) Simulated activation

(c) Reconstructed map

THANK U!!!